Energy conservation for fluid flows in an Onsager critical class

Abstract

The incompressible Euler equations govern the evolution of an ideal fluid. It is well known that the total kinetic energy is preserved along the time evolution of a regular fluid flow. However, when the motion is very rough, there is theoretical and experimental evidence of formation of chaotic structures that support the dissipation of kinetic energy. Mathematically, this problem translates into finding the critical regularity for weak solutions to the incompressible Euler equations to have conservation or dissipation of kinetic energy (Onsager’s conjecture). Currently, the Onsager conjecture is almost solved. It has been been proved that energy is conserved in any subcritical class and there are examples of solutions in any supercritical class violating the energy conservation. In a joint paper with Luigi De Rosa, we gave the first proof of energy conservation for weak solutions to the incompressible Euler system in a critical space, both in absence and presence of physical boundary. This is the first energy conservation result that holds in the incompressible case and fails in the compressible setting.