Bern-Fribourg Graduate Seminar

a seminar for Master and PhD students

Thursday 28th September, 2023: 17:15 - 18:00

Room B6, Exakte Wissenschaften, Bern

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An Application of Haar Measures to Statistical Mechanics

Abstract

In 1933, two papers appeared in succession in Annals of Mathematics, one by Alfred Haar and one by John von Neumann. In the former, Haar proved that any compact topological group admits a uniform measure - a probability measure μ such that if A and B are related by the group operations, then $\mu[A] = \mu[B]$. In the latter, von Neumann used the existence of this measure to answer Hilbert's 5th problem in the positive in the compact case. Since then, Haar measures have played a big role in the theory of topological groups, for instance by generalising the Fourier transform to allow for harmonic analysis on general, compact groups.

In this talk, we shall consider the group $\{0,1\}^E$, where E is the edge set of a graph $G = (V, E)$, where the group operation is pointwise addition mod 2. This group can be identified with the space of spanning subgraphs of G with group composition given by symmetric difference of edge sets. A closed subgroup thereof is given by the group of even subgraphs of G - that is, those subgraphs with even degree at every vertex. We will study the Haar measure on this space and compare it to other statistical mechanics models to illustrate some of its surprising properties. Finally, we shall show how to combine this with a coupling due to Grimmett and Janson in order to investigate the Ising model.