Fri-Seminar Studies plan

Responsible: Leonardo F. Cavenaghi

1. Goals

This is a course of seminars where students should prepare a written content working as a guide to their seminars. A list of proposed subjects is given below. These are separated for areas and work as suggestions. References are also given. If you have an idea of what presenting that is not listed, just let me know and we discuss the possibilities.

The written file and the presentations can be made in either French or English. If it is the case that the student struggle with giving talks in either of such languages, that can be made in Germany being only required the written file in English.

2. List of subjects

2.1. Analysis

1. Compute the n-volume of a n-ball of radius R. Show that this is given by the expression
$$V_n(R) = \frac{\pi^{n/2}R^n}{\Gamma(n/2+1)},$$
where $\Gamma(n)$ is the classical *Gamma function*, which coincides with the factorial function for $n \in \mathbb{N}$. Following the discussion here stated https://mathoverflow.net/questions/53119/volumes-of-n-balls-what-is-so-special-about-n-5v discuss what is special about the case $n = 5$. Also study the asymptotic behavior of the volume function as $n \to \infty$ and discuss the results (see [Par13]).
2. Construct a *Peano curve*, that is, a continuous surjection from $[0,1]$ to $[0,1]^2$ (see [Cav, Chapter 2]);
3. (i) Prove the *Baire's Cathegory theorem*: in a complete metric space a countable intersection of open and dense subsets is open and dense. (ii) Show that the set of somewhere differentiable functions on an interval

is meager in the fat space of continuous functions on the same interval views as a metric space with the supremum norm (see [Neu, Chapter 6.6, p.65]).

4. Prove the *Banach's fix point theorem* and study some of it applications to fractals (see [Neu, Chapter 6, Theorem 6.16, p.68]).

2.2. Number Theory

1. **The Carmichael function**. Consider the question: If $m = p.q$, where p, q are distinct primes, which is the maximum value of k such that there is an element over Z_m with order k? The answer is given by the so called *Carmichael function*. The proposal is to discuss such concept in the light of *primitive roots* (see [Ge]);
2. **Mersenne primes**. A *Mersenne prime* is a prime number that is one less than a power of two. That is, it is a prime number of the form $M_n = 2n - 1$ for some integer n. A *perfect number* is a positive integer that is the sum of all of its proper divisors, e.g, $6 = 1 + 2 + 3$. We propose: prove that all even perfect numbers is of the form $2^{p-1}M_p$, where p is a prime and M_p is a Mersenne prime. See the discussion https://math.stackexchange.com/questions/1528725/mersenne-primes-all-even-and-perfect-number-has-the-form-2p-1m-p-for-some.

2.3. Probability theory

This is a short topic chosen to promote a general understanding of some basics concepts in Probability Theory. The proposal is:

1. Enunciate and prove the *Birthday's Paradox Theorem* (see [Neu, Chapter 9, p. 95]);
2. Quickly recall the following statistics concepts: Baye's theorem, Expected value, Variance and the law of large numbers;
3. Discuss the following applications: (i) Buffon's needle (see [Neu, Chapter 9.3, p. 96]), (ii) prove the *Infinite Monkey Theorem*: a monkey hitting keys at random on a typewriter keyboard for an infinite amount of time will almost surely type any given text, such as the complete works of William Shakespeare.

3. What to expect from and how to prepare for this course

It is expected that every student attend to the talk of their colleagues. I am going to have meetings with the student that shall give the seminar on a certain week to help him to prepare the content. Apart from that, you are more than welcome to contact me anytime to discuss math or anything you may wish.

References

[Cav] David Cavender. Special-Filling Curves and Generalizing the Peano Curve.

[Ge] Yimin Ge. A note on the Carmichael Function.

[Neu] Jörg Neunhäuserer. 12^2 beautiful mathematical theorems with short proofs.

[Par13] Harold R. Parks. The volume of the unit n-ball. *Mathematics Magazine*, 86(4):270–274, 2013.

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