

Oberseminar Geometrie	Wednesday 11th December 2013
Department of Mathematics	10:20–12:00
University of Fribourg	Seminar room, Math. II (Lonza)

DANIELE VALTORTA (EPFL)

## ‘Quantitative stratification and applications to harmonic functions.’

The main objective of this talk is to present the quantitative stratification technique. In order to do so, we use the application to the critical sets of harmonic function as a guiding example.

The quantitative stratification is a technique based on the analysis of the symmetries of the object under study at different points in its domain, and their interactions with each other. It was introduced by Prof. Cheeger and Naber.

In detail, for harmonic functions we prove effective estimates on the volume of the tubular neighborhood of the critical set of  $u : B_1(0) \rightarrow \mathbb{R}$  of the form:

$$\text{Vol}(B_r(\mathcal{C}(u)) \cap B_{1/2}(0)) < C(n, \bar{N}^u(0, 1), \epsilon) r^{2-\epsilon}, \quad (1)$$

where  $\epsilon > 0$  is arbitrary, and  $\bar{N}^u$  is Almgren’s frequency function.

Given a harmonic function  $u$ , the standard *first order* stratification  $\{\mathcal{S}^k\}$  of  $u$  separates points  $x$  based on the degrees of symmetry of the leading order polynomial of  $u - u(x)$ . In this talk we introduce a quantitative stratification  $\{\mathcal{S}_{\eta,r}^k\}$  of  $u$ , which separates points based on the number of *almost* symmetries of *approximate* leading order polynomials of  $u$  at various scales.

By studying how nearby symmetric points interact, we prove an effective cone splitting theorem, and the monotonicity of the frequency function  $\bar{N}^u(r)$  will allow us to bound the number of scales where a function is not close to be symmetric. With these tools available, we use a simple covering argument to prove volume estimates on the tubular neighborhoods of  $\mathcal{S}_{\eta,r}^k$ . The estimates on the critical sets follow as a simple corollary.

Up to some technical detail, these techniques can be also adapted to prove estimates on the solutions to more general elliptic PDEs.