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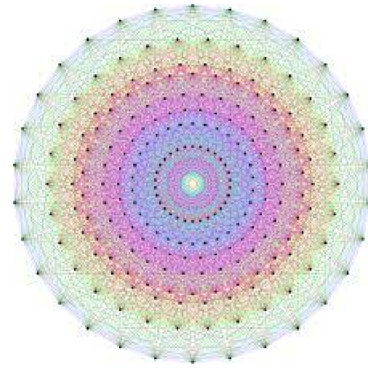

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Horizontally polynomial functions on Lie groups

In their proof of nonexistence of biLipschitz embeddings of the sub-Riemannian Heisenberg group $\mathbb{H}^1$ into L^1, Cheeger and Kleiner classify the precisely monotone sets in $\mathbb{H}^1$. A set E is *precisely monotone* if both E and E^c are h-convex; a set E is *h-convex* when, whenever a couple of points in E is joined by an integral curve of a horizontal vector field, then such a curve is entirely contained in E.

In arbitrary connected Lie groups $\mathbb{G}$ endowed with the choice of a set S of bracket generating directions in the Lie algebra $\mathfrak{g}$ of $\mathbb{G}$ - which we call *horizontal directions* - the study of a subclass of precisely monotone sets, namely the class of those sets that are sublevel sets of horizontally affine maps, naturally leads to the study of maps that are horizontally polynomial. A map (or more in general a distribution) f on a connected Lie group $\mathbb{G}$ is said to be *horizontally polynomial* if whenever X is in S, there exists k such that $X^k f \equiv 0$.

With a geometric argument we will show that on an arbitrary connected Lie group, the horizontal polynomials of bounded degree form a finite dimensional vector space. Moreover, for connected nilpotent Lie groups, we show that horizontally polynomial distributions are precisely those maps that are polynomial in exponential coordinates. This is a joint work with E. Le Donne.