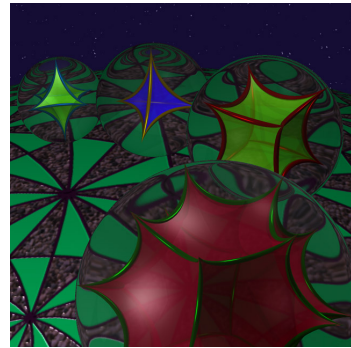


Oberseminar Geometrie
Department of Mathematics
University of Fribourg
Seminar room, Math II (Lonza)
Wednesday May 9, 2018, 10:20-12:00



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Aspherical circle bundles and a problem of Hopf

A long-standing question of Hopf asks whether every self-map of absolute degree one of a closed oriented manifold is a homotopy equivalence. This question gave rise to several other problems, most notably whether the fundamental groups of aspherical manifolds are Hopfian, i.e. any surjective endomorphism is an isomorphism. Recall that the Borel conjecture states that any homotopy equivalence between two closed aspherical manifolds is homotopic to a homeomorphism.

In this talk, we verify a strong version of Hopf's problem for certain aspherical manifolds. Namely, we show that every self-map of non-zero degree of a circle bundle over a closed oriented aspherical manifold with hyperbolic fundamental group (e.g. negatively curved manifold) is either homotopic to a homeomorphism or homotopic to a non-trivial covering and the bundle is trivial. Our main result is that a non-trivial circle bundle over a closed oriented aspherical manifold with hyperbolic fundamental group does not admit self-maps of absolute degree greater than one. This extends in all dimensions the case of circle bundles over closed hyperbolic surfaces (which was shown by Brooks and Goldman) and provides the first examples (beyond dimension three) of non-vanishing semi-norms on the fundamental classes of circle bundles over aspherical manifolds with hyperbolic fundamental groups.