Discretization and Spectrum of Laplacian

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Abstract

In order to understand the spectrum of Laplacian (acting on functions) associated to a compact Riemannian manifold, it may be powerful to discretize the manifold. The manifolds we will be interested in are the compact Riemannian manifolds with Ricci curvature and injectivity radius bounded below.

In the first lecture, we will define discretizations associated to such manifolds and show that the spectrum of the Laplacian on manifolds can be compared (in an uniform way) to the spectrum of the combinatorial Laplacian on discretizations. We then give an application of this result to a theorem of R. Brooks on the first eigenvalue of a tower of coverings.

In the second lecture, we will generalize the discretization to flat vector bundles. A vector bundle endowed with a metric compatible connection inherits naturally of a Laplacian called rough Laplacian. It turns out that the spectrum of the rough Laplacian can be compared to the spectrum of a discrete magnetic Laplacian. We then show with our technique that the holonomy of the vector bundle is related to the first eigenvalue of the rough Laplacian (this is a result of Ballmann-Brüning-Carron).