

Geometry of curve complexes and hyperbolic 3-manifolds

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In this course, we plan to give an overview of some of the recent advances in the theory of curve complexes and related structures associated to a compact surface.

The curve complex was introduced by Harvey [H] and has been studied by many authors since (see for example [I]). There are many connections with the theory of Teichmüller space, and the study of the mapping class groups. One of the principle motivations has been the work of of Minsky and his coworkers, notably Masur, Brock and Canary towards proving Thurston's ending lamination conjecture [Mi, BrCM]. A key result is that of Masur and Minsky [MaM] that tells us that the curve complex is hyperbolic in the sense of Gromov. To fully exploit this, however, one needs more information regarding certain hierarchical finiteness properties [MaM2].

We shall review some of this work, and its connections to the theory of hyperbolic 3-manifolds. We describe how some ideas from 3-manifold theory can be fed back into the study of curve complexes, for example, to understand the dynamics of the action of the mapping class group on the curve complex. Some of this work is surveyed in [Bo].

References

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