Summer School on Bounded Cohomology,

Coxeter Groups and Hyperbolic Geometry

August 18 – 20, 2008, University of Fribourg

commonweb.unifr.ch/math/events/SummerSchool08/SummerSchool08.html

Physics Department, room 2.52, Pérolles

Monday, August 18

09:30 - 10:30

Michelle BUCHER-KARLSSON: *Bounded cohomology and hyperbolic geometry I*

Coffee break

11:00 - 12:00

Anna FELIKSON: *Hyperbolic reflection groups I*

Lunch break

13:30 - 14:30

Michelle BUCHER-KARLSSON: *Bounded cohomology and hyperbolic geometry II*

Coffee break

15:00 - 16:00

Pavel TUMARKIN: *Reflection groups and Coxeter groups*

16:15 - 17:15 *Problem discussion*

Tuesday, August 19

09:30 - 10:30

Anna FELIKSON: *Hyperbolic reflection groups II*

Coffee break

11:00 - 12:00

Michelle BUCHER-KARLSSON: *Bounded cohomology and hyperbolic geometry III*

Lunch break

13:30 - 14:30

Anna FELIKSON: *Hyperbolic reflection groups III*

Coffee break

15:00 – 16:00

Michelle BUCHER-KARLSSON*: Bounded cohomology and hyperbolic geometry IV*

16:15 – 17:15 *Problem discussion*

17:30 – 18:30 *Reception in the Pavillon Vert of the Botanical Garden (see map)*

Wednesday, August 20

10:00 - ca. 19:00 *Excursion (see information webpage)*

Abstracts

Michelle Bucher-Karlsson:

Bounded cohomology is a relatively young theory introduced by Gromov in 1982. It differs from ordinary singular or group cohomology in that instead of considering arbitrary cochains, one restricts to cochains which have finite supremum norm. As a result, bounded cohomology behaves very differently from ordinary cohomology. A fundamental difference is that the bounded cohomology of a (reasonable) topological space is isomorphic to the bounded cohomology of its fundamental group. As a consequence, topological problems translate into questions about the fundamental group, such as understanding its space of representations. In this subject, one topological invariant of particular interest for its connections with geometry is the simplicial volume of closed oriented manifolds. For example, for Riemannian manifolds, it is proportional to the volume by a constant depending only on the universal covering, a phenomenon which is familiar from Hirzebruch's proportionality principle. In particular, it follows at once that the volume of Riemannian manifolds is a topological invariant. In general, simplicial volume computations, and a fortiori the determination of bounded cohomology groups, are very difficult. However, one instance where much can be said is for hyperbolic manifolds. Therefore, in this minicourse, after giving the first definitions and examples, I will concentrate on powerful applications of the theory to hyperbolic geometry. In particular I will discuss Milnor-Wood inequalities and nonexistence of affine structures on hyperbolic surfaces and products of hyperbolic surfaces, the proportionality principle for the simplicial volume for hyperbolic manifolds, and a simple geometric proof due to Gromov of Mostow's Rigidity Theorem for hyperbolic manifolds of dimension greater or equal than 3.

Anna Felikson:

Reflection groups are beautiful groups which arise naturally as symmetry groups of various mathematical objects as well as everyday life objects and art pieces. By a reflection group we mean a group generated by reflections with respect to some hyperplanes (or with respect to mirrors). The most interesting ones are the reflection groups acting discretely on a (symmetric) space, that means that the images of the mirrors never accumulate around any points of the space. This type of groups is a point where the interests of a child watching a kaleidoscope meet the interests of a scientist studying Riemmanian manifolds, or automorphic forms, or hypergeometric functions, or orbifolds, or arithmetics of quadratic forms, or K3 surfaces, etc. In this minicourse we will use regular polytopes to discuss general properties of reflection groups and Coxeter groups. We will consider discrete reflection groups in spherical and Euclidean spaces, and then we will focus on reflection groups acting discretely on a hyperbolic space. This space is really rich by examples of infinite reflection groups. On the other hand, hyperbolic reflection groups are far from being classified. We will show some classical results concerning hyperbolic reflection groups as well as describe some recent progress in this field. No special knowledge is required!

Pavel Tumarkin:

This lecture is an addendum to the course ``Hyperbolic reflection groups". We will discuss the relation between refelction groups and Coxeter groups. We will look at the combinatorial properties of Coxeter groups and see that any reflection group is a Coxeter group, and that any Coxeter group acts by reflections on some space with good properties.