The mini courses are intended to provide an introduction to the fields to Ph.D. students as well as postdocs working in nearby areas.

## Mini course by Professor Ngaiming Mok
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### Title: Holomorphic maps on bounded symmetric domains of rank $\geq 2$: Ergodic Theory, bounded holomorphic functions and geometric structures

**Abstract:** Over the years the author has been interested in rigidity problems on bounded symmetric domains of rank $\geq 2$. In this mini-course we give an overview on rigidity problems arising from *holomorphic mappings* either on bounded symmetric domains of rank $\geq 2$ or on their finite-volume quotient manifolds into complex manifolds.

The author's work concerning rigidity of holomorphic maps started off with a proof of Hermitian metric rigidity, i.e., uniqueness theorems on Hermitian metrics of nonpositive curvature in the sense of Griffiths on quotients $X := \Omega/\Gamma$ of bounded symmetric domains of rank $\geq 2$ by irreducible lattices (1987), a result which was completed in the case of nonuniform lattices by the work of Wing-Keung To (1989) using Satake-Baily-Borel compactifications. In the locally irreducible case this yields as a consequence that any nonconstant holomorphic map from $X$ into a Kähler manifold of nonpositive holomorphic bisectional curvarture must necessarily be an isometric immersion up to a normalizing constant. Another direction of research, initiated by the author and I-Hsun Tsai (1992) and by Tsai (1994), concerns holomorphic maps on irreducible bounded symmetric domains of rank $\geq 2$ themselves, in which case the assumption of equivariance with respect to a lattice is replaced by an assumption of properness on the map. We note a conceptual link between the two problems in that a holomorphic map $f : X \to X'$ between two compact quotients $X = \Omega/\Gamma$, $X' = \Omega'/\Gamma'$ lifts to a proper holomorphic map $F : \Omega \to \Omega'$ provided that the the induced map $f_* : \Gamma \to \Gamma'$ on fundamental groups is injective.

In this mini-course, after explaining some background we will focus on recent developments. We will highlight the use of some fundamental elements in the theory, including Ergodic Theory, bounded holomorphic functions and geometric structures. To start with, by studying foliations on minimal characteristic bundles and using Moore's Ergodicity Theorem for lattices on semisimple Lie groups we extend metric rigidity to the context of continuous complex Finsler metrics on $X = \Omega/\Gamma$ of nonpositive curvature in the generalized sense, obtaining a form of Finsler metric rigidity (2002). The latter is applied to the Carathédory pseudometric and a new related intrinsic pseudometric, through which we bring bounded holomorphic functions into the picture. This concerns especially bounded holomorphic functions which are in some sense extremal in relation to the lifted holomorphic map $F : \Omega \to \Omega'$. These results enlarge the scope of study of rigidity phenomena on holomorphic maps equivariant with respect to a lattice, allowing the target manifolds to be arbitrary bounded domains (2004). The study of boundary values of bounded holomorphic functions on bounded symmetric domains of rank $\geq 2$ also plays an important role. In fact, the integral representation of boundary values of bounded holomorphic functions by means

of Fatou's Lemma, used in conjunction with Ergodic Theory, allows us recently to give a function-theoretic proof of the same results with strengthened applications. At the same time, the same tool in Harmonic Analysis enables us to recover proper holomorphic maps from admissible limits on boundary components, and the approach is now linked in rigidity problems with the study of geometric structures, more specifically with the geometric theory of varieties of minimal rational tangents (VMRTs) that the author has been developing with Jun-Muk Hwang in the study of uniruled projective manifolds in Algebraic Geometry. Most recently, Jaehyun Hong and the author (2008) has established a rigidity result for germs of holomorphic maps between certain rational homogeneous spaces provided that they respect VMRTs.