Oberseminar Geometrie

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Thurston maps with four postcritical points

A Thurston map is a branched covering map of the 2-sphere which is not a homeomorphism and for which every critical point has a finite orbit under iteration of the map. Frequently, a Thurston map admits a description in purely combinatorialtopological terms. In this context it is an interesting question whether a given map can (in a suitable sense) be realized by a rational map with the same combinatorics. This question was answered by Thurston in the 1980's in his celebrated characterization of rational maps. Thurston's Theorem roughly says that a Thurston map is realized if and only if it does not admit a Thurston obstruction, which is an invariant multicurve that satisfies a certain growth condition. However, in practice it can be very hard to verify whether a given map has no Thurston obstruction, because, in principle, one would need to check the growth condition for infinitely many curves.

In this talk, we will consider a specific family of Thurston maps with four postcritical points that arises from Schwarz reflections on flapped pillows. Using a counting argument, we will prove a very simple necessary and sufficient condition for a map in this family to be realized by a rational map. In the last part of the talk, we will discuss a generalization of this result which states that, given an obstructed Thurston map with four postcritical points, one can eliminate obstructions by applying a so-called blowing up operation. (These results are joint with M. Bonk and M. Hlushchanka.)