Oberseminar Geometrie
Department of Mathematics
University of Fribourg
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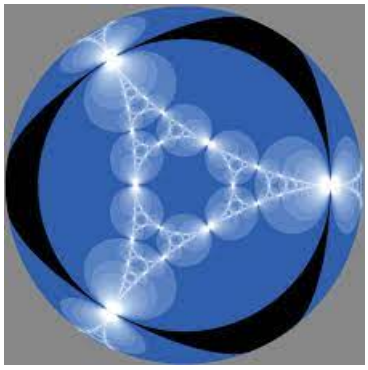

Mathias Blaise (UniFR)

On isometric invariants of cusped hyperbolic manifolds

Since the development initiated by Thurston, hyperbolic geometry plays a fundamental role in dimension three and beyond. In this work we consider cusped hyperbolic manifolds, that is, non-compact Riemannian manifolds of constant curvature -1, and we give bounds for three isometric invariants for such manifolds: the volume, the radius of the biggest embedded ball (inradius) and the length of a shortest closed geodesic (systole). The lower volume bound is a result of Meyerhoff, Adams and Kellerhals, wheras the bounds for the inradius and the systole are due to Gendulphe.

One particular 3-manifold stands out in this situation: the Gieseking manifold M_*. Constructed originally by face-pairing isometries of an ideal regular tetrahedron, the Gieseking manifold M_* is the unique cusped hyperbolic 3-manifold of minimal volume, up to isometry. Furthermore, we show that M_* realizes both the inradius bound and the systolic bound. While the systolic upper bound is attained by M_* and the (orientable) figure-eight knot complement, only, it is not yet known if the inradius bound characterizes M_* up to isometry.

Important tools and techniques in this work are densities of (horo-)ball packings in Euclidean and hyperbolic space, explicit formulae for the simplicial density function, hyperbolic trigonometry and various bounds for the dilation length of loxodromic isometries.