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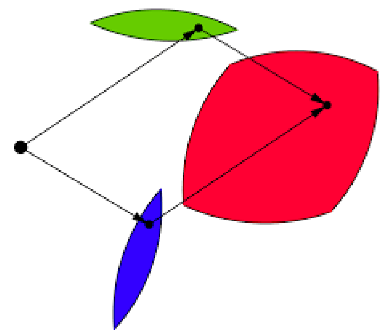

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Minkowski valuations

Let $\mathcal{K}^n$ denote the space of convex bodies (compact and convex sets) in $\mathbb{R}^n$. An operator $\Phi : \mathcal{K}^n \longrightarrow \mathcal{K}^n$ is called a Minkowski valuation if it satisfies

$$\Phi(K) + \Phi(L) = \Phi(K \cup L) + \Phi(K \cap L)$$

for every $K, L \in \mathcal{K}^n$ such that $K \cup L \in \mathcal{K}^n$. Here $+$ denotes the Minkowski sum, i.e., $A + B = \{a + b : a \in A, b \in B\}$, $A, B \subset \mathbb{R}^n$.

The difference body and the projection body operators, two classical operators in convex geometry, are examples of Minkowski valuations. The difference body operator can be easily defined as $DK = K + (-K)$, $K \in \mathcal{K}^n$.

First results about Minkowski valuations date back to the 70's when Schneider studied those satisfying some natural properties such as commuting with Euclidean motions. Nowadays, Minkowski valuations are much studied in convex geometry since, for instance, they proved to be useful to obtain new sharp analytical inequalities (such as generalizations of the Sobolev inequality).

After a brief historical account with the most fundamental results in Minkowski valuations, we will present characterization results (obtained together with Andrea Colesanti and Eugenia Saorín) for Minkowski valuations that satisfy the following volume constraint: there are constants $c, C > 0$ such that

$$c\,\mathrm{vol}(K) \leq \mathrm{vol}(\Phi(K)) \leq C\mathrm{vol}(K).$$

This volume constraint is satisfied by the difference body operator and, for it, it is known as the Rogers-Shephard inequality.

We will see that only two families of Minkowski valuations satisfy the volume constraint condition (together with continuity and translation invariance): a family of operators having as image cylinders over an $(n-1)$-dimensional convex body, and a family consisting essentially of 1-homogeneous operators. This result will imply new classification results for the difference body operator.